

**STM RC ACADEMY**

**DEPARTMENT OF MATHEMATICS**

**Introduction to A level Maths (and Further Maths)**

**GCSE Skills Revision**

## INTRODUCTION TO A LEVEL MATHS AT STM

The Mathematics A Level is a linear specification. There will be no examinations in Year 12 apart from school tests and mock examinations. The examinations for this course will be at the end of Year 13 and will consist of 3 two-hour papers. You will study pure mathematics, statistics and mechanics and will focus on reasoning and problem solving, modelling and the use of large data sets. The Mathematics Department is committed to ensuring that you make good progress throughout your Mathematics course. In order that you make the best possible start to the course, we have prepared this booklet.

It is *vitally important* that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics *before* you commence your course in September. You should have met all the topics before at GCSE. Work through the introduction to each chapter, making sure that you understand the examples. Then tackle the exercises. The answers are given at the back of the booklet and you should mark your own work highlighting any topics that you need more help with.

We will test you at the start of September to check how well you understand these topics, so it is important that you have looked at everything in the booklet before then. If you do not pass this test, you will be provided with a programme of additional work in order to bring your basic algebra skills to the required standard. You will then be re-tested in October. A mock test is provided at the back of this booklet.

We hope that you will use this introduction to give you a good start to your AS work and that it will help you enjoy and benefit from the course more.

**Mrs K Douglas Head  
of Mathematics**

Sources for further help are indicated throughout the booklet. You may also find the following online resource useful.

[OCR Bridging the Gap between GCSE and AS/A Level  
Mathematics](#)

Search for “[OCR Bridging the Gap between GCSE and AS/A Level  
Mathematics](#)” on google

[Hegarty Maths Youtube Live Lessons](#)

(Recordings)

Search for “[A-Level prep with Mr Hegarty](#)” on youtube

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## Chapter 1: REMOVING BRACKETS

To remove a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

### Examples

$$1) 3(x + 2y) = 3x + 6y$$

$$2) -2(2x - 3) = (-2)(2x) + (-2)(-3) \\ = -4x + 6$$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

\* the smiley face method \* FOIL  
(Fronts Outers Inners Lasts) \* using a grid.

### Examples:

$$1) (x + 1)(x + 2) = x(x + 2) + 1(x + 2)$$

or

$$(x+1)(x+2) = x^2 + 2 + 2x + x \\ = x^2 + 3x + 2$$

or

$$(x+1)(x+2) = \begin{array}{r} x \quad 1 \\ x \quad x^2 \\ x \quad 2 \\ 2x \quad 2 \end{array} = x^2 + 3x + 2$$

$$2) (x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3) \\ = 2x^2 + 3x - 4x - 6 = 2x^2 - x - 6$$

or

$$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$$

or

$$\begin{array}{r} x-2 \quad 2x \quad 2x^2-4x \quad 3 \\ \quad \quad \quad 3x-6 \end{array}$$

$$\begin{aligned} (2x+3)(x-2) &= 2x^2 + 3x - 4x - 6 \\ &= 2x^2 - x - 6 \end{aligned}$$

**EXERCISE A** Multiply out the following brackets and simplify.

1.  $7(4x + 5)$

2.  $-3(5x - 7)$

3.  $5a - 4(3a - 1)$

4.  $4y + y(2 + 3y)$

5.  $-3x - (x + 4)$

6.  $5(2x - 1) - (3x - 4)$

7.  $(x + 2)(x + 3)$

8.  $(t - 5)(t - 2)$

9.  $(2x + 3y)(3x - 4y)$

10.  $4(x - 2)(x + 3)$

11.  $(2y - 1)(2y + 1)$

12.  $(3 + 5x)(4 - x)$

**Two Special Cases**

**Perfect Square: Difference of two squares:**  $(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$   $(x - a)(x + a) = x^2 - a^2$   
 $(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$   $(x - 3)(x + 3) = x^2 - 3^2$   
 $= x^2 - 9$

**EXERCISE B** Multiply out

1.  $(x - 1)^2$
2.  $(3x + 5)^2$
3.  $(7x - 2)^2$
4.  $(x + 2)(x - 2)$
5.  $(3x + 1)(3x - 1)$
6.  $(5y - 3)(5y + 3)$

**More help** on expanding brackets is available by clicking on the video link at:  
<http://www.mathcentre.ac.uk/students/topics/algebra/brackets/>

**Chapter 2: LINEAR EQUATIONS**

When solving an equation, you must remember that whatever you do to one side must also be done to the other. The following operations are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain x<sup>2</sup> or x<sup>3</sup> terms.

**More help** on solving equations can be obtained by downloading the leaflet available at this website:

<http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simplelinear.pdf> Alternatively a video explanation is available by clicking on the video link at:

<http://www.mathcentre.ac.uk/students/topics/algebra/linearequations/>

**Example 1:** Solve the equation  $64 - 3x = 25$

**Solution:** There are various ways to solve this equation. One approach is as follows:

Step 1: Add  $3x$  to both sides (so that the  $x$  term is positive):  $64 = 3x + 25$

Step 2: Subtract 25 from both sides:  $39 = 3x$

Step 3: Divide both sides by 3:  $13 = x$

So the solution is  $x = 13$ .

**Example 2:** Solve the equation  $6x + 7 = 5 - 2x$ .

**Solution:** Step 1: Begin by adding  $2x$  to both sides  $8x + 7 = 5$  (to ensure that the  $x$  terms are together on the same side)

Step 2: Subtract 7 from each side:  $8x = -2$

Step 3: Divide each side by 8:  $x = -\frac{1}{4}$

**Exercise A:** Solve the following equations, showing each step in your working:

1)  $2x + 5 = 19$  2)  $5x - 2 = 13$  3)  $11 - 4x = 5$

4)  $5 - 7x = -9$  5)  $11 + 3x = 8 - 2x$  6)  $7x + 2 = 4x - 5$

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**Example 3:** Solve the equation  $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets:  $6x - 4 = 20 - 3x - 6$  (taking care of the negative signs)

Step 2: Simplify the right hand side:  $6x - 4 = 14 - 3x$

Step 3: Add  $3x$  to each side:  $9x - 4 = 14$

Step 4: Add 4:  $9x = 18$

Step 5: Divide by 9:  $x = 2$

**Exercise B:** Solve the following equations.

1)  $5(2x - 4) = 4$  2)  $4(2 - x) = 3(x - 9)$

3)  $8 - (x + 3) = 4$  4)  $14 - 3(2x + 3) = 2$

## EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

**Example 4:** Solve the equation  $5\frac{11}{2}y + 1 = 11$

**Solution:** Step 1: Multiply through by 2 (the denominator in the fraction):  $10\frac{11}{2}y + 2 = 22$

Step 2: Subtract 10:  $y = 12$

**Example 5:** Solve the equation  $\frac{1}{2}(2x + 1) + 5 = 3x + 1$

**Solution:** Step 1: Multiply by 2 (to remove the fraction)  $2\frac{1}{2}(2x + 1) + 10 = 6x + 2$

Step 2: Subtract 1 from each side  $2x = 14$

Step 3: Divide by 2  $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will remove both fractions.

**Example 6:** Solve the equation  $x + \frac{1}{4} + x + \frac{1}{5}$

**Solution:** Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator  $20\left(x + \frac{1}{4}\right) + 20\left(x + \frac{1}{5}\right)$

$= 20\left(x + \frac{1}{4}\right) + 20\left(x + \frac{1}{5}\right)$  Step 3: Simplify the left hand side

$$+ 20\left(x + \frac{1}{5}\right)$$

$$= 40\left(x + \frac{1}{4}\right) + 4\left(x + \frac{1}{5}\right) = 40$$

Step 4: Multiply out the brackets:  $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation:  $9x + 13 = 40$

Step 6: Subtract 13  $9x = 27$

Step 7: Divide by 9:  $x = 3$

**Example 7:** Solve the equation  $x + x - 4 = 2$

**Solution:** The lowest number that 4 and 6 go into is 12. So we multiply every term by 12

$$12(x + x - 4) = 12(2)$$

Simplify  $12x + 12x - 48 = 24 - 48$  Expand brackets  $12x + 12x - 48 = 24 - 48 + 24$

$24x - 48 = 24$  Subtract  $24x - 48 - 48 = 24 - 48 - 48$  Add 48  $24x = 72$  Divide by 24  $x = 3$

**Exercise C:** Solve these equations

1)  $2(x + 3) = 5$  2)  $3x - 1 = 3$

$x + 4$  3)  $4y + 3 = 5 - 3$

4)  $x - 7 = 2 +$

3 14

- x

8

**Exercise C (continued)**

5)  $7x - 1 = 13 - x$  6)  $y - 1 + y - 3 + 1 =$

$2y - 6$

+ 5

7)  $2x + x - 1 = 5x - 3$

+ 8)  $2 - 5x = 10x - 1$  **FORMING EQUATIONS**

**Example 8:** Find three consecutive numbers so that their sum is 96.

**Solution:** Let the first number be  $n$ , then the second is  $n + 1$  and the third is  $n + 2$ . Therefore  $n + (n + 1) + (n + 2) = 96$

$$3n + 3 = 96 \quad 3n = 93 \quad n = 31 \quad \text{So the numbers are } 31, 32, 33$$

**Exercise D:**

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has. Form an equation, letting  $n$  be the number of photographs one girl had at the **beginning**. Hence find how many each has **now**.

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### Chapter 3: SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is  $3x + 2y = 8$  1  $5x + y = 11$  2

In these equations,  $x$  and  $y$  stand for two numbers. We can solve these equations in order to find the values of  $x$  and  $y$  by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate  $y$ . We do this by making the coefficients of  $y$  the same in both equations. This can be achieved by multiplying equation 2 by 2, so that both equations contain  $2y$ :

$$3x + 2y = 8 \quad 1 \quad 10x + 2y = 22 \quad 2 \times 2 = 3$$

To eliminate the  $y$  terms, we subtract equation 3 from equation 1. We get:  $7x = 14$

$$\text{i.e. } x = 2$$

To find  $y$ , we substitute  $x = 2$  into one of the original equations. For example if we put it into 2:

$$10 + y = 11$$

$$y = 1 \quad \text{Therefore the solution is } x = 2, y = 1.$$

**Remember:** You can check your solutions by substituting both  $x$  and  $y$  into the original equations.

**Example:** Solve  $2x + 5y = 16$  1  $3x - 4y = 1$  2

**Solution:** We begin by getting the same number of  $x$  or  $y$  appearing in both equations. We can get  $20y$  in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$$8x + 20y = 64 \quad 3 \quad 15x - 20y = 5 \quad 4$$

As the SIGNS in front of  $20y$  are DIFFERENT, we can eliminate the  $y$  terms from the equations by ADDING

$$23x = 69 \quad 3+4 \text{ i.e. } x = 3$$

Substituting this into equation 1 gives:

$$6 + 5y = 16 \quad 5y = 10 \text{ So... } y = 2 \text{ The solution is } x = 3, y = 2.$$

If you need **more help** on solving simultaneous equations, you can download a booklet from the following

<http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simultaneous1.pdf> Alternati

video explanation is available by clicking on the video links at:

<http://www.mathcentre.ac.uk/students/topics/algebra/simultaneous-equations/>

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### Exercise:

Solve the pairs of simultaneous equations in the following questions:

1)  $x + 2y = 7$  2)  $x + 3y = 0$

$$3x + 2y = 9 \quad 3x + 2y = -7$$

3)  $3x - 2y = 4$  4)  $9x - 2y = 25$

$$2x + 3y = -6 \quad 4x - 5y = 7$$

$$5) 4a + 3b = 22 \quad 6) 3p + 3q = 15 \quad 5a - 4b = 43 \quad 2p + 5q = 14$$

## Chapter 4: FACTORISING

### Common factors

We can factorise some expressions by taking out a common factor.

**Example 1:** Factorise  $12x - 30$

**Solution:** 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

**Example 2:** Factorise  $6x^2 - 2xy$

**Solution:** 2 is a common factor to both 6 and 2. Both terms also contain an  $x$ .

So we factorise by taking  $2x$  outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

**Example 3:** Factorise  $9x^3y^2 - 18x^2y$

**Solution:** 9 is a common factor to both 9 and 18.

The highest power of  $x$  that is present in both expressions is  $x^2$ . There is also a  $y$  present in both parts. So we factorise by taking  $9x^2y$  outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

**Example 4:** Factorise  $3x(2x - 1) - 4(2x - 1)$

**Solution:** There is a common bracket as a factor.

So we factorise by taking  $(2x - 1)$  out as a factor. The expression factorises to  $(2x - 1)(3x - 4)$

### Exercise A

Factorise each of the following

1)  $3x + xy$

2)  $4x^2 - 2xy$

3)  $pq^2 - p^2q$

4)  $3pq - 9q^2$

5)  $2x^3 - 6x^2$

6)  $8a^5b^2 - 12a^3b^4$

7)  $5y(y - 1) + 3(y - 1)$

### Factorising quadratics

**Simple quadratics: Factorising quadratics of the form  $ax^2 + bx + c$**  The method is: Step 1: Form two brackets  $(\dots)(x \dots)$  Step 2: Find two numbers that multiply to give  $c$  and add to make  $b$ . These two numbers get written on the other end of the brackets.

**Example 1:** Factorise  $x^2 - 9x - 10$ .

**Solution:** We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1. Therefore  $x^2 - 9x - 10 = (x - 10)(x + 1)$ .

**General quadratics: Factorising quadratics of the form  $ax^2 + bx + c$**  The method is: Step 1: Find two numbers that multiply together to make  $ac$  and add to make  $b$ . Step 2: Split up the  $bx$  term using the numbers found in step 1. Step 3: Factorise the front and back pair of expressions as fully as possible. Step 4: There should be a common bracket. Take this out as a common factor.

**Example 2:** Factorise  $6x^2 + x - 12$ .

**Solution:** We need to find two numbers that multiply to make  $6 \times -12 = -72$  and add to make 1. These two numbers are -8 and 9.

Therefore,  $6x^2 + x - 12 = 6x^2 - 8x + 9x - 12$

$$= 2x(3x - 4) + 3(3x - 4) \text{ (the two brackets must be identical)}$$

$$= (3x - 4)(2x + 3)$$

**Difference of two squares: Factorising quadratics of the form  $x^2 - a^2$**

Remember that  $x^2 - a^2 = (x + a)(x - a)$ .

Therefore:  $9x^2 - 16 = (3x + 4)(3x - 4)$

$$16x^2 - 25 = (4x + 5)(4x - 5)$$

Also notice that:  $8x^2 - 4 = 2(4x + 2)(4x - 2)$  and  $3x^2 - 48 = 3(x + 4)(x - 4)$

**Factorising by pairing** We can factorise expressions like  $2x^2 + xy - 12x - 6y$  using the method of factorising by pairing:

$$2x^2 + xy - 12x - 6y = x(2x + y) - 6(2x + y) \text{ (factorise front and back pairs, ensuring both brackets are identical)} = (2x + y)(x - 6)$$

If you need **more help** with factorising, you can download a booklet from this website:

<http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf>

Alternatively, a video explanation is available by clicking on the video links at:  
<http://www.mathcentre.ac.uk/students/topics/algebra/factor-quadratics/>

### Exercise B

Factorise

1)  $6x^2 - x - 1$

2)  $6x^2 + 16x - 6$

3)  $2x^2 + 5x + 2$

4)  $3x^2 - 23x + 3$  (factorise by taking out a common factor)

5)  $3x^2 + 5x - 2$

6)  $2y^2 + 17y + 21$

7)  $7y^2 + 10y - 3$

8)  $10x^2 + 5x - 30$

9)  $4x^2 - 25$

10)  $3x^2 + 3xy - 3y^2$

11)  $4x^2 + 12x + 8$

12)  $16m^2 - 81n^2$

13)  $3^2 4 9 y a y -$

14)  $^2 8( 1) 2( 1) 10 x x + - + -$

### Chapter 5: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation, we must do the same to both sides in order to keep the equation balanced.

**Example 1:** Make  $x$  the subject of the formula  $y = 4x + 3$ .

**Solution:**  $y = 4x + 3$  Subtract 3 from both sides:  $y - 3 = 4x$  Divide both sides by 4;  $\frac{y - 3}{4} = x$

$\frac{y - 3}{4} = x$  So  $x = \frac{y - 3}{4}$

$\frac{y - 3}{4}$

is the same equation but with  $x$  the subject.

**Example 2:** Make  $x$  the subject of  $y = 2 - 5x$

**Solution:** Notice that in this formula the  $x$  term is negative.

$y = 2 - 5x$  Add  $5x$  to both sides  $y + 5x = 2$  (the  $x$  term is now positive) Subtract  $y$  from both sides  $5x = 2 - y$

Divide both sides by 5  $x = \frac{2 - y}{5}$

$\frac{2 - y}{5}$

**Example 3:** The formula  $C = \frac{5}{9}(F - 32)$

is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make  $F$  the subject.

$C = \frac{5}{9}(F - 32)$

Multiply by 9  $9C = 5(F - 32)$  (this removes the fraction) Expand the brackets  $9C = 5F - 160$

both sides  $9C + 160 = 5F$  Divide both sides by 5  $\frac{9C + 160}{5} = F$

$\frac{9C + 160}{5}$

Therefore the required rearrangement is  $F = \frac{9C + 160}{5}$

$\frac{9C + 160}{5}$

### Exercise A

Make  $x$  the subject of each of these formulae:

1)  $y = 7x - 1$  2)  $y = \frac{x + 4}{5}$

3)  $4y = 3x - 2$  4)  $y = \frac{4(3^x - 9) - 5}{15}$

### Rearranging equations involving squares and square roots

**Example 4:** Make  $x$  the subject of  $x^2 + y^2 = w^2$  **Solution:**  $x^2 + y^2 = w^2$  Subtract  $y^2$  from both sides:  $x^2 = w^2 - y^2$  (this isolates the term involving  $x$ )

Square root both sides:  $x = \pm \sqrt{w^2 - y^2}$  Remember that you can have a positive or a negative square root. Simplify the answer any more.

**Example 5:** Make  $a$  the subject of the formula  $4t = a + h$

$4t = a + h$

Multiply by  $-1$ :  $-4t = -a - h$  Square both sides:  $16t^2 = a^2 + h^2$  Multiply by  $h$ :  $16t^2h = a^2h + h^3$  Divide by  $h$ :

$16t^2h = a^2 + h^3$  **Exercise B:**

Make  $t$  the subject of each of the following

1)  $P = 32wt + r$  2)  $P = \frac{wt}{32r}$

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### More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In such questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

**Example 6:** Make  $t$  the subject of the formula  $a - xt = b + yt$  **Solution:**  $a - xt = b + yt$  Start by collecting  $t$  on the right hand side: Add  $xt$  to both sides:  $a = b + yt + xt$  Now put the terms without a  $t$  on the left hand side:

Subtract  $b$  from both sides:  $a - b = yt + xt$  Factorise the RHS:  $a - b = t(y + x)$  Divide by  $(y + x)$ :  $\frac{a - b}{y + x} = t$

$x = t$  So the required equation is

**Example 7:** Make  $W$  the subject of the formula  $T - W = Wa + 2b$  **Solution:** This formula is complicated by a fractional term. We begin by removing the fraction: Multiply by  $2b$ :  $2bT - 2bW = Wa + 2b$  Add  $2bW$  to both sides:  $2bT = Wa + 2b + 2bW$  (this collects the  $W$ 's together) Factorise the RHS:  $2bT = W(a + 2b)$  Divide both sides by  $a + 2b$

$W = \frac{2bT}{a + 2b}$  If you need more help you can download an information booklet on rearranging equations from the following website:

<http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-formulae2-tom.pdf> Alternatively, an explanation is available by clicking on the video links at:

<http://www.mathcentre.ac.uk/students/topics/algebra/transposition/> **Exercise C**

Make  $x$  the subject of these formulae:

1)  $ax + 3 = bx + c$  2)  $3(x + a) = k(x - 2)$  3)  $y = 5x^2$

$x - 2 = 3x^3$

4)  $a^x = 1 + b^x$

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## Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form  $ax^2 + bx + c = 0$ .

There are two methods that are commonly used for solving quadratic equations: \* factorising \* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used, however.

### Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of  $x^2$  is positive.

**Example 1 :** Solve  $x^2 - 3x + 2 = 0$

Factorise  $(x - 1)(x - 2) = 0$  Either  $(x - 1) = 0$  or  $(x - 2) = 0$  So the solutions are  $x = 1$  or  $x = 2$

Note: The individual values  $x = 1$  and  $x = 2$  are called the **roots** of the equation.

**Example 2:** Solve  $x^2 - 2x = 0$

Factorise:  $x(x - 2) = 0$  Either  $x = 0$  or  $(x - 2) = 0$  So  $x = 0$  or  $x = 2$

### Method 2: Using the formula

Recall that the roots of the quadratic equation  $ax^2 + bx + c = 0$  are given by the formula:

$x =$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a

**Example 3:** Solve the equation  $2x^2 - 5 = 7 - 3x$  **Solution:** First we rearrange so that the right hand side is zero to get  $2x^2 + 3x - 12 = 0$  We can then tell that  $a = 2$ ,  $b = 3$  and  $c = -12$ . Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$

(this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get:  $x = 1.81$  or  $x = -3.31$

If you need more help with the work in this chapter, there is an information booklet downloadable from the following site:

<http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-quadratic-equations.pdf> Alternative explanation is available by clicking on the video links at:

<http://www.mathcentre.ac.uk/students/topics/algebra/solving-quadratics/>

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## EXERCISE

1) Use factorisation to solve the following equations: a)  $x^2 + 3x + 2 = 0$  b)  $x^2 - 3x - 4 = 0$

c)  $x^2 = 15 - 2x$

2) Find the roots of the following equations: a)  $x^2 + 3x = 0$  b)  $x^2 - 4x = 0$

c)  $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a)  $6x^2 - 5x - 4 = 0$  b)  $8x^2 - 24x + 10 = 0$



- 2)  $3c^2 \times 2c^5 =$
  - 3)  $bc^2 \times bc^3 =$
  - 4)  $2n^6 \times (-6n^2) =$
  - 5)  $8n^8 \div 2n^3 =$
  - 6)  $d^{11} \div d^9 =$
  - 7)  $(a_3)^2 =$
  - 8)  $(d_4)^3 =$
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**More complex powers**

**Zero index:** Recall from GCSE that

$a_0 = 1$ . This result is true for any non-zero number  $a$ .

Therefore  $5_0 = 1$

$( ) 3 4$

$( ) = 1 - 5.2304_0 = 1$  **Negative powers**

A power of -1 corresponds to the reciprocal of a number, i.e.  $a^{-1} =$

$\frac{1}{a}$  Therefore  $5^{-1} = \frac{1}{5}$

$0.25^{-1} = 0.25$

$1 = 4 \left( \frac{1}{5} \right) 4$

(you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers:  $a^{-n} =$

$\frac{1}{a^n}$

$n$ . This means:

$3^{-2} =$

$\frac{1}{3^2} = \frac{1}{9}$   $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$   $4^{-1} = \frac{1}{4}$  **Fractional powers:** Fractional powers correspond to roots:

$a^{1/2} = \sqrt{a}$   $a^{1/3} = \sqrt[3]{a}$   $a^{1/4} = \sqrt[4]{a}$  In general:  $a^{1/n} = \sqrt[n]{a}$  Therefore:  $8^{1/3} = \sqrt[3]{8} = 2$   $25^{1/2} = \sqrt{25} = 5$   $10000^{1/4} = \sqrt[4]{10000} = 10$

A more general fractional power can be dealt with in the following way:  $a^{m/n} = (\sqrt[n]{a})^m$  So  $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

$$2 = \left( \left( \left( \begin{matrix} 2 & 3 \\ 36 & 25 \end{matrix} \right) \right)^{2/3} \right)^{3/2} = 9 \cdot 4$$

$$3 = \left( \left( \left( \begin{matrix} 6 & 5 \end{matrix} \right) \right)^{3/2} \right)^{-3/2} = 125 \cdot 216$$

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### Exercise B:

Find the value of:

$$1) 4^{1/2} \cdot 27^{1/3} \cdot 3 \left( 19 \right)^{1/2} \cdot 4 \cdot 5^{-2} \cdot 5 \cdot 18^0 \cdot 6 \cdot 7^{-1} \cdot 7 \cdot 27^{2/3} \cdot 8$$

$$2) \left( \left( \left( \begin{matrix} 9 \\ 8 \end{matrix} \right) \right)^{-2/3} \cdot 10 \right) \left( \begin{matrix} 0.04 \\ 8 \end{matrix} \right)^{1/2}$$

$$1) \left( \left( \left( \begin{matrix} 1 \\ 1 \end{matrix} \right) \right)^{-3/2} \right) \text{Simplify each of the following:}$$

13)  $2 a^{1/2} \times 3 a^{5/2}$  14)  $x^3 \times x^{-2}$  15)  $(x^2 y^4)^{1/2}$  If you need more help with the work in this chapter, there is information booklet downloadable from this web site:

<http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-indicespowers-2009-1.pdf> Alternatively, a video explanation is available by clicking on the video links at:

<http://www.mathcentre.ac.uk/students/topics/algebra/powers/>

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## Chapter 8: FRACTIONS

### Adding and subtracting

In order to add or subtract fractions, they have to have the same denominator (bottom number). Once case, adding and subtracting are easy.

Here is a typical question.

**Example 1:** Work out  $\frac{1}{5} + \frac{1}{2}$

**2. Solution:** We need both top and bottom numbers to be the same. We need to find the smallest number both 5 and 2 can divide into - that is, the **lowest common multiple** of 5 and 2. This is 10. So we change denominators into 10: Remember to multiply both the denominator (bottom number) and numerator (top number) by the same number.

$$\frac{1}{5} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{2}{2 \cdot 5} = \frac{2}{10}$$

$$\frac{1}{5} = \frac{2}{2 \cdot 5} = \frac{2}{10}$$

$$\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{2}{10} = \frac{4}{10} = \frac{2}{5}$$

$$+ 10^5$$

$$= 10^7$$

The common denominator is not always the two denominators multiplied together (for instance, the lowest common multiple of 4 and 6 is **12**) but you can use this as a rule - it might be that you have to cancel your answer at the end.

**Example 2:** Work out  $\frac{7}{9} - \frac{1}{6}$

**16Solution:** The lowest common multiple of 9 and 6 is **18**, but we could use  $9 \times 6 = 54$ . Here are both ways showing that we get the same answer:

Using 18

$$\frac{7}{9} - \frac{1}{6}$$

$$= \frac{14}{18} - \frac{3}{18}$$

$$= \frac{14 - 3}{18}$$

$$= \frac{11}{18}$$

$$= \frac{11}{18}$$

$$= \frac{11}{18}$$

Using 54

$$\frac{7}{9} - \frac{1}{6}$$

$$= \frac{49}{54} - \frac{9}{54}$$

$$= \frac{49 - 9}{54}$$

$$= \frac{40}{54}$$

$$= \frac{40}{54}$$

$$= \frac{40}{54}$$

But  $\frac{40}{54}$  can be cancelled to  $\frac{20}{27}$  by dividing the top and bottom by 2:

$$\frac{40}{54} = \frac{20}{27}$$

354 333

$$\frac{20}{27} = \frac{20}{27}$$

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**Fractions and whole numbers**

$2\frac{3}{4}$  is a **mixed number**. **2** is a whole number and  $\frac{3}{4}$  is a **fraction**. You can change this into a top heavy fraction by writing  $\frac{34}{4}$

$$= 11$$

4. To change a top heavy fraction into a mixed number you first divide the top of the fraction (**numerator**) by the bottom (**denominator**).  $\frac{13}{4} = 3\frac{1}{4}$  because 4 divides into 13 three times with one left over.

### **Adding and Subtracting fractions with mixed numbers**

Remember that to add fractions you must make the denominators equal.

**Example 3:** Calculate

$$2\frac{2}{3} + 1\frac{5}{5}$$

$$2$$

$$2\frac{3}{3} +$$

$$1\frac{4}{5} = \frac{8}{5}$$

$$+ 1\frac{5}{5}$$

$$=$$

$1\frac{567}{415} = 4\frac{15}{7}$  Or, alternatively, you can also do this by adding the numbers and then the fractions. Use the method you prefer.

$$2$$

$$2\frac{3}{3} +$$

$$1\frac{4}{5} = 3 + \frac{2}{3} + \frac{4}{5} =$$

$$3$$

$$+ \frac{10}{15} + \frac{12}{15} =$$

$$4\frac{15}{7} = 3$$

$$+ 1\frac{5}{5}$$

$$24$$

### **Multiplying**

Multiplying fractions is very easy - we just **multiply the numerators** (top numbers) and **multiply the denominators** (bottom numbers). This will probably mean that you can cancel down the answer at the end. You can do this before you start.

You can try to find common factors between any of the numerators and any of the denominators. You can cancel down diagonally - they don't have to belong to the same fraction.

**Example 4:** Work out

$$5^2 \times$$

**Solution:** We can cancel the top 2 with the bottom 8, thus  $2 \times 7$

8 becomes this  $1 \times 5$

$$7^4 = 2401$$

### Multiplying with mixed numbers

You will sometimes need to multiply fractions with mixed numbers. You **must** always remember to turn mixed numbers into top heavy fractions before you multiply.

### Dividing

The basic rule when dividing 2 fractions, is to turn the 2nd fraction upside down, and multiply. The reciprocal of a number is 1 divided by that number. For instance, to work out the reciprocal of 4: We can think of 4 as  $\frac{4}{1}$  turned upside down is  $\frac{1}{4}$ . The reciprocal of a fraction is the fraction turned upside down. So the reciprocal of  $\frac{2}{3}$  is  $\frac{3}{2}$ .

So to work out  $\frac{2}{3} \div \frac{3}{4}$

we work out  $\frac{2}{3} \times \frac{4}{3}$

$$\frac{8}{9}$$

9.

**Remember** We only turn the fraction we are dividing by (the second one) upside down.

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**Dividing mixed numbers** Again, turn any mixed numbers into top heavy fractions at the start. Then turn the fraction you are dividing by and multiply.

**Example 5:** Work out

3

13 ÷

$$2 \frac{2}{3} = \frac{10}{3}$$

$3 \div \frac{10}{3}$  **Solution:**

=

$$3 \times \frac{3}{10} = \frac{9}{10}$$

3 ÷

$$= \frac{8}{10} \times 8^3$$

### =EXERCISE

Work out the following:

(a)  $5^2_3 + 63_4$  (b)  $63_4 - 5^2_3$  (c)  $5^2_3 \times 63_4$  (d)  $5^2_3 \div 63_4$

If you need more help with the work in this chapter, there is an information booklet downloadable from the site:

<http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-fracbas>

<http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-fracac>

<http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-fracmult-2009-1.pdf> Alternatively, a video is available by clicking on the

<http://www.mathcentre.ac.uk/students/topics/arithmetric/fractions/>

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## Practice Booklet Test

You may NOT use a calculator

If  $ax^2 + bx + c = 0$  then  $x = -$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Expand and simplify

(a)  $(2x + 3)(2x - 1)$  (b)  $(a + 3)^2$  (c)  $4x(3x - 2) - x(2x + 5)$

2. Factorise

(a)  $x^2 - 7x$  (b)  $y^2 - 64$  (c)  $2x^2 + 5x - 3$  (d)  $6t^2 - 13t + 5$

3. Simplify

(a)  $\frac{8yx^3}{ac^2}$

(b)  $\frac{23x + 3}{14x - 6}$

(c)  $\frac{23x + 3}{14x - 6}$

(d)  $\frac{23x + 3}{14x - 6}$

(e)  $\frac{23x + 3}{14x - 6}$

(f)  $\frac{23x + 3}{14x - 6}$

4. Solve the following equations

(a)  $h = 4$

(b)  $x^2 - 8x = 0$  (c)  $p^2 + 4p = 12$

5. Write each of the following as single powers of  $x$  and  $y$

(a)  $1x^4$  (b)  $(x^2y)^3$  (c)  $x^5x^{-2}$  6. Work out the values of the following, giving your answers as fractions

1 (a)  $4^{-2}$  (b)  $10^0$  (c)

$$\left( \frac{1}{27} \right)^8$$

7. Solve the simultaneous equations  $3x - 5y = -11$   $5x - 2y = 7$

8. Rearrange the following equations to make  $x$  the subject

(a)  $v^2 = u^2 + 2ax$  (b)  $V = 1/3\pi x^2h$  (c)  $y = x + 2x + 19$ . Solve  $5x^2 - x - 1 = 0$  giving your solutions in surd form

out the following:  
(a)  $2\sqrt{2} + 3\sqrt{3}$  (b)  $3\sqrt{4} - 2\sqrt{2}$  (c)  $2\sqrt{2} \times 3\sqrt{3}$  (d)  $2\sqrt{2} \div 3\sqrt{3}$

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## SOLUTIONS TO THE EXERCISES

**CHAPTER 1:** Ex A 1)  $28x + 35$  2)  $-15x + 21$  3)  $-7a + 4$  4)  $6y + 3y^2$  5)  $2x - 4$  6)  $7x - 1$  7)  $x^2 + 5x + 6$  8)  $t^2 - 3t - 1$   
 $12y^2$  10)  $4x^2 + 4x - 24$  11)  $4y^2 - 1$  12)  $12 + 17x - 5x^2$  Ex B 1)  $x^2 - 2x + 1$  2)  $9x^2 + 30x + 25$  3)  $49x^2 - 28x + 4$  4)  $x^2 - 4x + 4$   
 6)  $25y^2 - 9$

**CHAPTER 2** Ex A 1) 7 2) 3 3)  $1\frac{1}{2}$  4) 2 5)  $-\frac{3}{5}$  6)  $-\frac{7}{3}$  Ex B 1) 2.4 2) 5 3) 1 4)  $\frac{1}{2}$  Ex C 1) 7 2) 15 3)  $\frac{24}{7}$  4)  $\frac{35}{3}$   
 $\frac{9}{5}$  8) 5 Ex D 1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

**CHAPTER 3** 1)  $x = 1, y = 3$  2)  $x = -3, y = 1$  3)  $x = 0, y = -2$  4)  $x = 3, y = 1$  5)  $a = 7, b = -26$   $p = \frac{11}{3}, q = \frac{4}{3}$

**CHAPTER 4** Ex A 1)  $x(3 + y)$  2)  $2x(2x - y)$  3)  $pq(q - p)$  4)  $3q(p - 3q)$  5)  $2x^2(x - 3)$  6)  $4a^3b^2(2a^2 - 3b^2)$  7)  $(y - 1)(5y - 3)$   
 $(x - 3)(x + 2)$  2)  $(x + 8)(x - 2)$  3)  $(2x + 1)(x + 2)$  4)  $x(2x - 3y)$  5)  $(3x - 1)(x + 2)$  6)  $(2y + 3)(y + 7)$  7)  $(7y - 3)(y - 1)$   
 $+ 2$  9)  $(2x + 5)(2x - 5)$  10)  $(x - 3)(x - y)$  11)  $4(x - 2)(x - 1)$  12)  $(4m - 9n)(4m + 9n)$  13)  $y(2y - 3a)(2y + 3a)$  14)  $2(x - 1)(x + 2)$

**CHAPTER 5** Ex A 1)  $x = \frac{y + 1}{7}$

2)  $x = 4y - 5$  3)  $x = 3(4y + 2)$  4)  $x =$

$$\frac{9}{y} - 12$$

$$+ \frac{20}{y}$$

Ex B 1)  $t = \frac{32rP}{w}$  2)  $t = \pm \frac{32rP}{w}$  3)  $t = \pm \frac{3V}{h\pi}$  4)

$t = \frac{P}{2g}$  5)  $t = \frac{v - Pa}{g}$  6)  $t = \pm \frac{r - a}{b}$  Ex C 1)  $x = \frac{a^c - b^3}{-3}$

$$2) x = \frac{3}{a} - k$$

$$+ \frac{2}{k} - 3$$

$$3) x = 5 \frac{2}{y}$$

$$y - \frac{2}{3}$$

4)  $x = \frac{ab}{a - b}$  **CHAPTER 6** 1) a) -1, -2 b) -1, 4 c) -5, 3 2) a) 0, -3 b) 0, 4 c) 2, -2 3) a)  $-\frac{1}{2}, \frac{4}{3}$  b) 0.5, 2.5 4) a)

1.07, -0.699 c) -1.20, 1.45 d) no solutions e) no solutions f) no solutions

**CHAPTER 7** Ex A 1)  $5b^6$  2)  $6c^7$  3)  $b^3c^4$  4)  $-12n^8$  5)  $4n^5$  6)  $d^2$  7)  $a^6$  8)  $-d^{12}$  Ex B 1) 2 2) 3 3)  $1/3$  4)  $1/25$  5) 1 6)  $1/7$  7)  $1/4$  10) 0.2 11)  $4/9$  12) 64 13)  $6a^3$  14)  $x$  15)  $xy^2$

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**CHAPTER 8** a)  $12^5_{12}$  b)  $11^1_{12}$  c)  $381/4$  d)  $1^{13}_{68}$

**SOLUTIONS TO PRACTICE BOOKLET TEST**

1) a)  $4x^2 + 4x - 3$  b)  $a^2 + 6a + 9$  c)  $10x^2 - 13x$

2) a)  $x(x - 7)$  b)  $(y + 8)(y - 8)$  c)  $(2x - 1)(x + 3)$  d)  $(3t - 5)(2t - 1)$

3) a)  $2_2 xy$  b)  $10 x +$

$^3 64$  a)  $h = 5$  b)  $x = 0$  or  $x = 8$  c)  $p = -6$  or  $p = 2$

5) a)  $x^4$  b)  $x^6 y^3$  c)  $x^7$

6) a)  $1_{16}$  b) 1 c)  $2_3$

7)  $x = 3, y = 4$

8) a)

$$x = \sqrt{v^2 - u^2}$$

2

ab)  $x = 3V h\pi$  c)  $x = 2$

$y - 1^y$  9)  $x =$

$1 \pm 10^2$  1

10) a)  $61/4$  b)  $11/4$  c)  $9^3_8$  d)  $2_3$

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